Analysis of Cascading Failures in Small-world Power Grid

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***Abstract*—The development of complex network theory, especially the discovery of small-world characteristics in power grids, has made people more and more concerned on the structural vulnerability of electric power system. On the basis of complex network theory, this paper studies the heterogeneity of small-world power grid, specially analyzes the inherent reason for cascading failures in small-world power grid and looks for valid measures to improve the capability of the power grid to endure large-scale blackouts. Failure simulations have been done on topological models of Anhui power grid and the East China power grid. The results show that structural vulnerability of small-world power grid is the intrinsic reason for failures to cascade in the grid and external factor can hardly improve its endurance to large-scale cascading failures. Meanwhile, strategies to prevent large-scale cascading failures are presented and tested, and it is found that more reasonable resource configuration and load distribution are meaningful to increase the reliability of electricity transmission.**

Keywords-small-world power grid; cascading failures; heterogeneity; structural vulnerability

I. INTRODUCTION

Over the last few years, large-scale cascading blackouts and the catastrophic damage have made people much more concerned on the security and reliability of power grid [1-2]. On one hand, researchers have comprehensively and systematically analyzed the current conditions of power grid: paper [3-6] studied the cumulative probability distribution of failures, simulated the self-organization process of the power grid and analyzed the influence of electricity supply and demand on the happening of large-scale blackouts. Paper [7-9] established different hidden failure models to simulate large-scale cascading blackout initiated by hidden failures of the protection system and analyzed the risk of hidden failures. To prevent the immense damage caused by cascading failures, those papers suggested that maintenance of protection system should be enforced and grid capacity be enlarged. On the other hand, study of structural vulnerability to find the intrinsic reason for cascading of failures has become more and more popular, and much meaningful results have been obtained: paper [10-17] proved that the west U.S. power grid, the North China power grid and the East China power grid are all small-world networks. Paper [11] qualitatively analyzed the influences of small-world characteristics on the enlargement of failures, indicating that the short average distance and high clustering coefficient have helped accelerate the spreading of failures. Paper [12] proved the dependency of small-world power grid on its key nodes. Based on paper [13, 14], this paper tries to find the inherent reason for cascading of failures in small-world power grid by studying its heterogeneity of nodes, endurance to different modes of failures and reactions to internal and external factors.

The structure of the paper is as below: in the 1st section, small-world characteristics are analyzed on the topological model of power grid. In the 2nd section, load distribution and degree distribution of small-world power grid are studied. The cascading failure model is introduced in section 3, failure simulations on real power grids are done in section 4. In the end, the paper analyzed the inherent reason for cascading failures in small-world power grid and provided corresponding measures to improve the reliability of power grid.

II. SMALL-WORLD CHARACTERISTICS OF POWER GRID

A. Small-world Network

In 1998, by reconnecting the edges of regular network with a small probability p, Watts established a new type of network. Compared to regular network with large clustering coefficient and average distance, and random network with small clustering coefficient and average distance, the new network which contains large clustering coefficient and small average distance simultaneously is called “small-world network”. The formulation process of small-world network is shown in fig. 1; its geometric character is [10]:

$$\begin{cases} C >> C_{random} \\ L \geq L_{random} \end{cases} \tag{1}$$

Where Crandom~K/n, Lrandom~ln(n)/ln(K). n is the total number of node, K is the average degree of the network, Crandom and Lrandom are the clustering coefficient and

average distance of the random network.

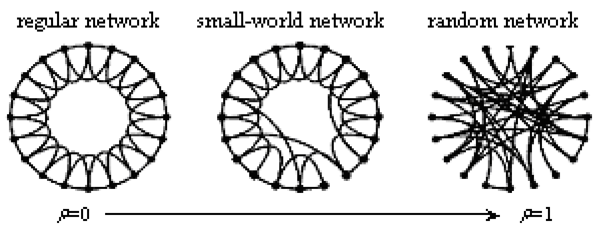

Fig. 1 Formulation process of small-world grid.

Note: the small-world network (middle) is formed by rewiring the edges in the regular network (left). Where, p is the probability for each edge to be rewired. For p=0, the network is regular, for p=1, it turns into a random network.

From the formulation process of small-world network, it is easy to find that the rewiring of edges has introduced a few long-edges (edges linking one node to 'far away' nodes) to the network so that information transmission is accelerated, resulting in variations of the geometric character [10]. Therefore, the long-edges are the most important part to affect the paths and efficiency of information transmission in small-world network.

B. Topological Model of Power grid

The power grid is generally represented as an idealized undirected, sparse and connected graph, in which, nodes are the generation plants or substations, edges are high voltage transmission lines, the graph is described by the n×n adjacency matrix｛eij｝[11].

In the topological model, the power grid runs in the best scenario: Firstly, nodes are classified into three types according to their functions in real power grid and positions in the topological model [13, 20]: (1) Generation nodes: These nodes are connected to generators via transformers in real power grid. They are the sources of electric energy. (2)Distribution nodes: No generators are connected to such nodes, and in the topological structure each node is connected to only one single edge. Electricity is sent to consumers from such distribution nodes through low voltage transmission lines. (3)Transmission nodes: No generators are connected to such nodes either, but in the topological structure each node is connected to more than one edge. Secondly, different from refs [17-21] where the electricity is transmitted through the most efficient way, the present paper applies the reactance of the transmission lines as entry of the adjacency matrix｛eij｝, and for parallel transmission lines, equivalent reactance is calculated, so that electricity is transmitted from generation nodes to all other reachable nodes through shortest electrical paths, which is more realistic than through the most efficient way. Thirdly, in the topological model, the capacity (load flow) of the power grid is determined by generation nodes and distribution nodes, but the efficiency of electricity transmission is determined by the transmission nodes because their existence ensures that the electricity is transmitted through the shortest electrical paths. Therefore, transmission nodes are the focus of electricity transmission efficiency and the number of shortest electrical paths traversing the transmission node defines its load.

C. Small-world Characteristics of Power Grid

Data of "the East China 2004 summer summit power flow of the operation side" is used to establish the topological models of Anhui power grid and the East China power grid, the average distance (in power grid, the average distance is the average length of shortest electrical paths) and clustering coefficient of the grids are calculated. The topological information and parameters are listed in table 1 and table 2.

TABLE I. TOPOLOGICAL INFORMATION OF THE POWER GRIDS

POWER GRIDS	n	E	N_G	N_L	N_T
ANHUI	84	112	16	14	54
EAST CHINA	769	1029	112	183	474

Note: E, N_G, N_L and N_T represent the number of total node, total edges, generation nodes, distribution nodes and transmission nodes respectively.

TABLE II. CHARACTERISTIC PARAMETERS OF THE POWER GRIDS

POWER GRIDS	K	C_{ACTUAL}	C_{RANDOM}	L_{ACTUAL}	L_{RANDOM}
ANHUI	2.670	0.107	0.032	6.597	4.518
EAST CHINA	2.676	0.088	0.004	11.791	8.063

Note: C_{actual} and L_{actual} are the clustering coefficient and average distance of real power grid.

From table 1 and table 2, though both Anhui grid and the East China grid are large power grids, their topological characters are not always the same: both the average distance and clustering coefficient of Anhui power grid are close to those of the random network. But compared with the random configuration, a slightly longer average distance and a much larger clustering coefficient are seen in the East China power grid. Therefore, it is reasonable to conclude that Anhui power grid is a non small-world network, the East China power grid is a typical small-world network.

III. DISTRIBUTION OF LOAD AND DEGREE IN SMALL-WORLD POWER GRID

A. Load Distribution of Small-world Power Grid

Since it's the transmission nodes that transmit electricity from generation nodes to electricity users, the load that a transmission node takes can reflect its status and function in

the whole power grid. Fig. 2-a and fig. 2-b show the logarithmic curves of load distribution for Anhui power grid and the East China power grid respectively.

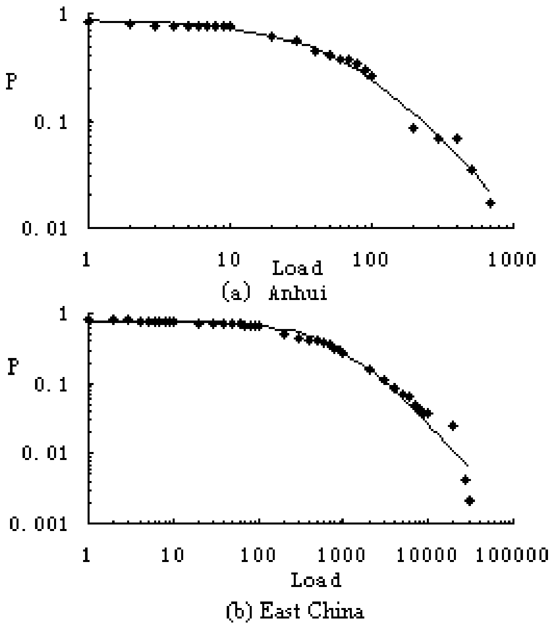

Fig. 2 Cumulative load distribution of the power grids.

From fig. 2, load distributions of the two grids both follow power law, but there are many subtle differences: firstly, the power law index is about -1.71 in Anhui power grid (fig. 2-a), and -1.48 in the East China power grid (fig. 2-b), which means that power law phenomenon in the East China power grid is more obvious than in Anhui power grid; secondly, load of Anhui power grid extends narrowly (from 0 to 396), nodes status is relatively equal in load; while in the East China power grid, nodes load covers a large span (from 0 to 30146), there are a few nodes taking much higher load than the rest nodes.

The difference in load distribution suggests that in small-world power grid, nodes status is heterogeneous in the process of electricity transmission. Anhui grid is rich for electric energy, the distances between generation nodes and distribution nodes are short, thus long-edges are unnecessary and transmission nodes take relatively equal load in the procedure of electricity transmission. However, as to the East China power grid that contains small-world characteristics, energy sources and users are distributed so unevenly that long-edges are formulated to enhance electricity transmission efficiency. Since the shortest electrical paths bypass those long-edges with priority, some of the nodes connected to long-edges have to take over much higher load than others. That is to say, it is the demand for transmitting electricity in the most efficient way that causes the power grid to contain small-world characteristics.

B. *Degree Distribution of Small-world Power Grid*

The degree of a node is the edges connected to it. Degree distributions of Anhui power grid and the East China power grid are shown in fig. 3.

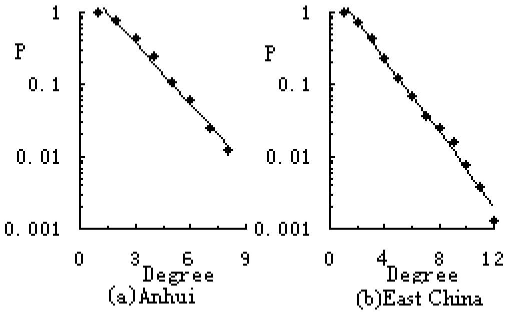

Fig. 3 Cumulative degree distribution of the power grids.

According to fig. 3, nodes degree covers 0~8 and 0~12 respectively in the two power grids, obvious linearity is seen in each of the logarithmic curves and the slopes are -0.65 and -0.58 respectively. These indicate that power grids are sparse networks, the connections among the nodes are relatively equal and small-world power grid is no exception.

IV. THE MODEL FOR SIMULATION OF CASCADING FAILURES

In this section, a cascading failure model is presented, in which, concepts of operating capacity and operating limit are introduced, the cascading procedure is simulated by dynamic distribution of nodes load, the loss of load is evaluated under different modes of initial failure.

A. *The Cascading Failure Model*

In the stable state, each of the nodes in the power grid runs within their operating capacity. When some nodes are faulted and exit from operation, the electricity transmission paths will be diverted, other nodes might take more load and run beyond their operating capacity or even the operating limit. To simulate the evolution of the power grid under disturbance and following the specialty of electricity transmission on the topological model, when the power grid is disturbed by nodes failure, the rest nodes load is calculated dynamically [14-20], reactance of relative transmission lines are regulated according to nodes operating capacity and operating limit so that the electrical paths are diverted, the overloaded nodes load is decreased and the cascading of failures slows down.

Suppose Li is the initial load for each node, a0 its operating capacity factor, a1 its operating limit factor, then the node operating capacity and operating limit are:

$$S_i = a_0 * L_i \tag{2}$$

$$S_{\max i} = a_1 * L_i \tag{3}$$

where Si is node's operating limit, Smaxi is node's operating

capacity.

Line reactance regulation follows three rules, they are:

$$e_{ij} = e_{ij0} \qquad (L_i < S_i) \tag{4}$$

$$e_{ij} = e_{ij0} * L_i / S_i \qquad (S_i < L_i < S_{\max i}) \tag{5}$$

$$e_{ij} = \infty \qquad (L_i \geq S_{\max i}) \tag{6}$$

where eij0 is the initial line reactance, eij is the regulated line reactance.

In other words, when the node runs within its operating capacity, reactance of the lines connected to it keeps the original value, when node load is larger than its operating capacity but lower than the operating limit, the reactance is regulated according to the overloading ratio, when the load exceeds its operating limit, the node exit from operation, corresponding line reactance becomes infinite. When all the nodes run within their operating capacity, the system enters a new stable status.

B. Failure Index

The loss of load is introduced to evaluate the scale of failure spreading. It is expressed in (7).

$$L_{cut} = \frac{\sum_{j \in G_1} L_j}{\sum_{k \in G_0} L_k} \times 100\% \tag{7}$$

where, G_1 is the collection of all transmission nodes that exit from operation, G_0 is the collection of all transmission nodes.

C. Failure Modes

Two failure modes are established to study the structural vulnerability of small-world power grid and external factors' influence on the reliability of power grid.

1) Failure mode 1

Mode 1 is designed to study the dependency of small-world power grid on high degree nodes and high load nodes. The experiments on this mode include two schemes, the initial faulted nodes in scheme (1-1) are chosen in the decreasing order of their degrees, the initial faulted nodes in scheme (1-2) are chosen in the decreasing order of their load. In each scheme, the number of the initial faulted nodes increases step by step.

2) Failure mode 2

Mode 2 is designed to study the reaction of power grid to internal and external factors. 2 schemes are included in the experiments of mode 2.

Because of the uncertainty of the distribution of long-edges in small-world network, transmission nodes are sorted into nT/10 groups according to the decreasing order of nodes load, the load of each group is the average load of the nodes in the group. In scheme (2-1), 20 simulations are done with the first 20 groups of nodes as the initial faulted nodes separately. In scheme (2-2), one group of nodes is randomly chosen as the initial faulted nodes, but the capacity factor increases with the pace of 0.0125.

V. SIMULATION AND ANALYSIS OF CASCADING FAILURES IN SMALL-WORLD POWER GRID

The paper testified the above failure model on both Anhui power grid and the East China power grid.

A. Structural Vulnerability of Small-world Power Grid

Under the two schemes of mode 1, a0 is set 1.1, a1 is set 1.5. Failure indexes of the two schemes in mode 1 are recorded to compare the influences of high degree nodes and high load nodes on power grid reliability, and the ratio of the index of scheme (1-1) to that of scheme (1-2) are calculated to particularly reflect the dependency of small-world power grid on high load nodes.

Results of the simulations on Anhui power grid and the East China power grid are shown in fig. 4 and fig. 5 respectively.

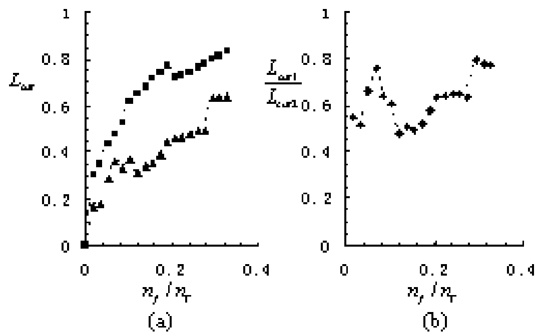

Fig. 4 Failure index variation of Anhui power grid in mode1.

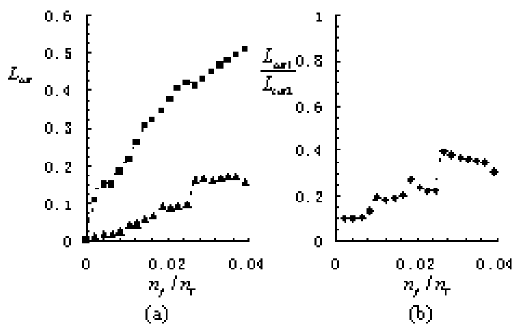

Fig. 5 Failure index variation of the East China power grid in mode1.

Note: in fig. 4-a, and fig. 5-a, triangular is the result of scheme (1-1), rectangular is that of scheme (1-2). n_f is the number of initial faulted

transmission nodes, n_T was demonstrated in Tab. 1. In fig. 4-b and fig. 5-b, $L_{cut1}$and L_{cut2} are failure indexes in scheme (1-1) and scheme (1-2) respectively.

For Anhui power grid, when nodes with high degree and high load are faulted, the consequence of scheme (1-1) is a little lighter than that of scheme (1-2) (fig. 4-a), failure index of scheme (1-1) is around 60% of scheme (1-2) though the ratio fluctuates a lot (fig. 4-b).

As to the East China power grid, when nodes with high degree are faulted, the failure index only rises a little at nf /nZ =0.02; but the failure index rises fast if only a few nodes with high load are faulted (nf /nZ =0.004) (fig. 5-a); besides, index of scheme (1-1) is around 25% of the index of scheme (1-2) and the ratio hardly fluctuates (fig. 5-b).

Comparing the results of the two grids, when nodes with high degree and high load are faulted, failures cascade in different ways in non small-world power grid and small-world power grid: firstly, if high degree nodes are faulted, failures spread slowly in both non small-world power grid and small-world power grid; secondly, compared with non small-world power grid, when the small part of nodes with high load are faulted, small-world power grid becomes so vulnerable that the shortest electrical paths divert a lot, causing the power transmission capacity to decrease much more than failures on nodes with high degree.

B. Internal and External Factors on Reliability of Small-world Power Grid

Generally speaking, decreasing the initial faulted nodes load and increasing the capacity factor are equal to weakening the disturbance on the power grid, Lcut should decrease. Therefore, for scheme (2-1) and scheme (2-2), the first simulation causes much heavier consequence than the rest simulations.

Suppose Lcut1 is the failure index in the 1st simulation, Lcuti the failure index in the ith simulation, then, the improvement of the power grid to endure failures can be expressed as:

$$L_{imp} = 1 - \frac{L_{cuti}}{L_{cut1}} \tag{8}$$

where, Limp is the improvement of the power grid to endure failures.

Fig. 6-a shows the improvement of failure endurance in Anhui power grid and the East China power grid when decreasing the initial faulted nodes load; Fig. 6-b shows the improvement of failure endurance of the two power grids when node capacity factor is increased.

From fig. 6-a, it is seen that the endurance capability of failures are increased in both Anhui power grid and the East China power grid, but the East China power grid shows a better condition. From fig. 6-b, it is seen that the failure endurance capability of Anhui power grid jumps about 70% when the load capacity factor is increased to a certain degree. However, even if the factor increases for 20 paces, the failure endurance capability of the East China power grid hardly improves.

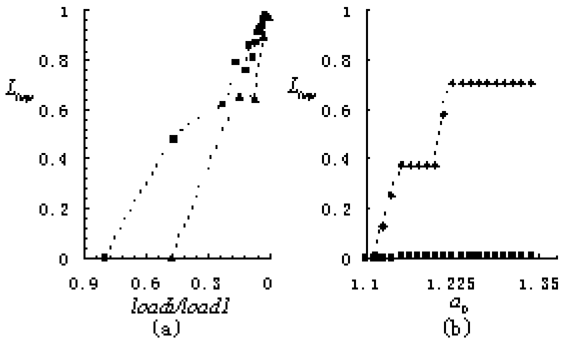

Fig. 6 Failure improvements by internal and external factors

Note: triangular is the simulation results of Anhui power grid, and rectangular is the results of the East China power grid. In fig. 5, $load_i$ and $load_1$ represent the average load of the nodes in the *i*th group and the *1*st group.

Comparing fig. 6-a and fig. 6-b, the two power grids react differently to internal and external factors: for non small-world power grid, to increase the grid capacity can largely improve its ability to endure cascading failures. But for small-world power grid, external factor can hardly improve its ability to endure cascading failures; improvement of the grid structure to decrease the concentration of load on certain nodes may have better effect on its capability to endure cascading failures.

C. Analysis of the Character of Cascading Failures in Small-world Power Grid

The two power grids studied in this paper contain different topological characteristics; their capabilities to endure cascading failures are also different: as to non small-world power grid, the failures develop similarly when different kinds of nodes are faulted and adequate enhancement of the grid capacity can slow down the development of failures. As to small-world power grid, when nodes with high load are faulted, the failure index swiftly jumps to a high degree, and external factor can hardly improve its capability to endure cascading failures. In other words, the consequences of cascading failures on small-world power grid is much heavier than on non small-world power grid, once the vulnerable parts of the grid are destroyed, common means can hardly prevent the cascading trend essentially.

The special topological structure of small-world power grid explains the essential reason for its vulnerability: nodes

of small-world power grid show strong heterogeneity in load, the existence of those long-edges, that help to increase the efficiency of electricity transmission, has made some of the nodes relative to them take over much higher load than other nodes. Meanwhile, the key nodes caused by long-edges are also the vulnerable parts of small-world power grid, when they are faulted and exit from operation, the shortest electrical paths divert a lot and failures spread swiftly in the grid. To effectively prevent the development of large-scale cascading failures and increase the reliability of small-world power grid, energy configuration must be improved to avoid the unevenness of load distribution in the grid.

VI. CONCLUSIONS

The paper specially studied the topological character of small-world power grid. Compared with non small-world power grid, the degree distribution of small-world power grid is not special, but its load distribution shows obvious power law, there are a few nodes taking over much higher load than other nodes, and their existence has increased the electricity transmission efficiency. Cascading failure simulation of small-world power grid indicates that, the nodes with high load caused by the long-edges in small-world power grid are the vulnerable part of the grid, once they are faulted and exit from operation, the immense diversion of transmission paths is the essential reason for the cascading of failures. To prevent the development of cascading failures and increase the operating reliability of small-world power grid, a more reasonable configuration of the power grid is more effective than singly enhancing the grid capacity.

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